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Two kinds of empirical truth and realism about idealized laws

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Abstrakt

Dwa rodzaje prawd empirycznych a realizm w kwestii praw idealizacyjnych

Z semantycznego punktu widzenia prawda empiryczna jest prawdziwością zdań, których terminy mają odniesienia empiryczne. Jednakże tylko niektóre takie odniesienia bezpośrednio i bez zniekształceń odnoszą się do rzeczywistych zjawisk; inne odnoszą się do nich poprzez ich idealizacje. W pierwszym przypadku mówimy o prawdzie opisowej, a w drugim o prawdzie przybliżonej. Niniejszy artykuł ma na celu rozjaśnienie tego rozróżnienia za pomocą pewnej wersji semantycznej teorii prawdy i poprzez odwołanie się do kontekstu debaty realizm/antyrealizm w kwestii praw idealizacyjnych. Artykuł składa się z dwóch części. Pierwsza nieformalnie wyjaśnia szczegóły tego rozróżnienia i krytykuje powszechne przekonanie, że wyidealizowane prawa lub związane z nimi założenia są fałszywe. Druga przedstawia formalno-semantyczne ujęcie, w którym wyidealizowane prawa są zarówno referencyjnie prawdziwe (w wyidealizowanym modelu), jak i w przybliżeniu prawdziwe (w odniesieniu do struktury docelowego systemu).

Słowa kluczowe: semantyczna teoria prawdy, prawda referencjalna, prawda przybliżona, idealizacja, prawda opisowa

Abstract

Two kinds of empirical truth, and realism about idealized laws

From the semantic point of view, empirical truth is the truth of sentences whose terms have empirical references. However, only some such references directly and without distortion relate to real-world phenomena, while others relate to them only through their idealizations. In the former case, we speak of descriptive truth, and in the latter, of approximate truth. This paper aims to clarify this distinction using a version of the semantic theory of truth, invoking the context of the realism/anti-realism debate about idealized laws. The paper consists of two parts. The first informally clarifies the details of the distinction and criticizes the widespread belief that idealized laws or the assumptions they involve, are falsehoods. The second sets out to a formal-semantic account in which the idealized laws are both referentially true (in the idealized model) and approximately true (of the target system's structure).

Keywords: semantic theory of truth, referential truth, approximate truth, idealization, descriptive truth

1. Introduction

Various concepts of truth are used today in the context of the dispute around scientific realism:

> Most people define scientific realism in terms of the truth or approximate truth of scientific theories […]. The scientific realist holds that science aims to produce true descriptions of things in the world (or approximately true descriptions, or ones whose central terms successfully refer, and so on).[1]

It seems that the following distinction between three basic concepts of truth will allow us to better grasp the conceptual framework underlying this dispute:

- *referential truth*, analyzable semantically in terms of *reference* — i.e. the intended interpretation of extralogical constants (both empirical and mathematical);
- *descriptive truth*, analyzable epistemologically in terms of *description* — i.e. the propositional presentation of real-world phenomena (in the sense of empirical facts);
- *approximate truth*, analyzable methodologically in terms of *idealization* — i.e. simplified scientific representations of empirical facts — or in terms of theoretical progress toward a true explanation of the world.

In subsequent decades, the general idea of *referential truth* has often been assumed within the logico-semantic context of the realism/antirealism debate (this being largely a reflection of Dummett and Putnam's employment of the logical principle of bivalence as a criterion for realism). Following Alfred Tarski's semantic theory of truth, we will identify it with the interpretation function of extralogical constants in the language's intended model.[2]

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1 A. Chakravartty, *Scientific realism*, in: *The Stanford encyclopedia of philosophy* (2017), ed. E. Zalta, https://plato.stanford.edu/archives/sum2017/entries/scientific-realism/.

2 Theo A. F. Kuipers used a similar concept in the context of the realism/antirealism debate: "According to referential realism, entity and attribute terms are intended to refer, and frequently we have good reasons to assume that they do or do not refer […]. Here, the referential truth is of course the strongest true referential claim which can be made by a certain vocabulary about a certain domain"

Meanwhile, the concept of *descriptive truth*, paired with the opposite idea of descriptive falsehood, has frequently been deployed by critics of scientific realism. Here are some representative examples of this, taken from the works of Nancy Cartwright and Catherine Elgin:

> The fundamental laws of physics do not describe true facts about reality. Rendered as descriptions of facts, they are false; amended to be true, they lose their fundamental, explanatory force.[3]

> Most scientific explanations use *ceteris paribus* laws. These laws, read literally as descriptive statements, are false.[4]

> Effective idealizations are felicitous falsehoods [...]. Nothing in the world exactly answers to them, so as descriptions, they are false.[5]

> We often convey information and advance understanding by means of sentences and other representations that are not literally true. An adequate epistemology should account for these as well.[6]

Some authors clarify "descriptive truth" in terms of possible worlds as truth in the actual world.[7] Here, by contrast, the terminology of the semantics of possible worlds will not be used. Instead, the framework of model theory will be used.

The phrase "approximate truth" is ambiguous; in discussions of scientific realism, it is most often used to mean two different (but related) things. The first can be clarified regarding theoretical explanations that are only partially

(T. A. F. Kuipers, *Epistemological positions in the light of truth approximation*, "The Paideia Archive: Twentieth World Congress of Philosophy" 37 [1998], p. 135). Richard M. Sainsbury discusses various properties often attributed to the reference relation. See R. M. Sainsbury, *The essence of reference*, in: *The Oxford handbook to the philosophy of language*, eds. E. Lepore, B. Smith, Oxford 2008.

3 N. Cartwright, *How the laws of physics lie*, Oxford 1983, p. 54.

4 N. Cartwright, *How the laws of physics lie*, p. 52.

5 C. Z. Elgin, *Understanding and the facts*, "Philosophical Studies" 132 (2007) no. 1, p. 39.

6 C. Z. Elgin, *True enough*, Cambridge 2017, p. 16.

7 Cf. T. A. F. Kuipers, *Approaching descriptive and theoretical truth*, "Erkenntnis" 18 (1982) no. 3, pp. 343–378.

true and open to future fundamental revision. The word "truthlikeness" is sometimes used to express this meaning. The second can be clarified in terms of idealized models that only approximate real phenomena;[8] the following quote provides a typical context for this use:

> While it is false that the other planets have no gravitational effect on Mars, and false as well that its orbit is elliptical, both of these statements are, in some sense, approximately true.[9]
>
> Many of the most plausible candidates for approximately true statements — such as "The Earth is spherical," "The acceleration of a body is directly proportional to the resultant force acting on it, when its speed is much lower than the speed of light," and Kepler's second law of planetary motion — are false.[10]

In this paper, only the second meaning is used. Its main task is to clarify the difference between descriptive and approximate truth using a referential version of the semantic theory of truth[11] supported by the following three observations:

- that empirical terms are potentially ambiguous in that they have, depending on the context, a descriptive or idealized reference, the latter being methodologically basic in the context of deliberate idealization;

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8 Cf. R. Hilpinen, *Approximate truth and truthlikeness*, in: *Formal methods in the methodology of empirical sciences*, eds. M. Przełęcki, K. Szaniawski, R. Wójcicki, Dordrecht–Boston–Wrocław 1974, p. 21.

9 T. Weston, *Approximate Truth and Scientific Realism*, "Philosophy of Science" 59 (1992) issue 1, p. 57.

10 D. P. Rowbottom, *Can meaningless statements be approximately true? On relaxing the semantic component of scientific realism*, "Philosophy of Science" 89 (2022) issue 5, p. 884.

11 The articles by M. Przełęcki (*The concept of truth in empirical languages*, "Grazer Philosophische Studien" 3 (1977) issue 1, pp. 1–17) and R. Wójcicki (*Theories, theoretical models, truth. Part II: Tarski's theory of truth and its relevance for the theory of science*, "Foundations of Science" 4 (1995/96), pp. 471–516), as well as the encyclopedia entries by W. Hodges (*Tarski's truth definitions*, in: E. Zalta, *The Stanford encyclopedia of philosophy* (2018), https://plato.stanford.edu/entries/tarski-truth/) and J. Woleński (*The semantic theory of truth*, in: *Internet encyclopedia of philosophy* (2021), https://iep.utm.edu/s-truth/), provide a general introduction to the semantic theory of truth. (In addition, the articles by Wójcicki and Przełęcki discuss some specific issues of the applications of this theory in the empirical sciences.) Section 5 will define a referential version of this theory as its model-theoretic version, in which the object language is part of the metalanguage.

- that the approximate truth of a target (system's) structure can be treated semantically as referential truth within the model that is an idealization of this structure;
- that the idealization relation can be defined as the converse of the non-empty concretization relation between idealized models and target structures.

The paper responds to the first two observations by clarifying the intuitive notion of *empirical truth* as truth that includes terms whose references can only be established by empirical methods (such as observation, measurement, idealization, etc.). The essence of this clarification is to identify the empirical truth of a given target structure with the referential truth of a sentence whose intended interpretation is represented either by that structure or by its idealization. In the former case, sentences are descriptively true, and in the latter case, they are approximately true. The two kinds of truth express two basic ways of semantically representing the target structure: by directly referring to it and through its idealization.[12]

The above observations, together with their semantic clarifications, will be used here in the context of the debate surrounding the realism about idealized laws (ILs), which holds that ILs are both referentially true (in a given idealized model) and approximately true (of a given target system's structure) according to their intended interpretation.[13]

The paper consists of two parts. The first (Sections 1–4) seeks to clarify in informal terms the distinction between two kinds of empirical reference (descriptive and idealized) within the context of the realism/antirealism about ILs debate (Sections 2 and 3), to shed light on realist and antirealist positions about ILs (Section 3), and to critique the antirealist interpretation of the fact that ILs, or their assumptions, are descriptively false (Section 4). The second

12 It is worth emphasizing that this distinction cannot be taken as an exemplification of *alethic pluralism*, proclaiming that different domains of discourse determine different properties of truth.

13 The works of L. Nowak (*Wstęp do idealizacyjnej teorii nauki*, Warszawa 1977; *The idealizational approach to science: a survey*, Poznań 1992, pp. 9–63) provide an extensive discussion of the idealization method, the nature of ILs, and the role of the process of their concretization in the progress of science. A more precise definition of realism about ILs will be formulated in the final part of Chapter 3.

(Sections 5–9) sets out to establish the formal framework of the semantic theory of referential truth (Section 5) and use it to show that ILs are referentially true (Section 6). It then extends this theory with the notion of idealization as a relation between idealized and descriptive models, in order to semantically clarify both the idea of descriptive truth (Section 7) and the realist belief that ILs are approximately true (Section 8). Finally, a perspective on the development of a theory of target-system representation will be outlined (Section 9).

2. The essential distinctions

The examples given below will, I hope, serve to better illuminate the ideas of referential and empirical truth, as well as the distinction between two kinds of the latter: namely, descriptive and approximate truth.

Referential truths are sentences that are true in the intended interpretations of their constants. For instance, the sentences "2+2=4" and "The empty set is included in every set" are referential truths because they are true in the intended interpretation of their mathematical constants: i.e. "2," "4," "+," "is a set," "empty set," and "is included." For a similar reason, the sentences "The Earth is not flat" and "The Earth orbits the Sun" are referential truths. Additionally, they are empirical because of the empirical interpretation of some of their terms (at least "Earth" and "Sun").

Descriptive truths directly and without distortion link their terms with the elements of target systems' structures. The sentences "The Earth is not flat" and "The Earth orbits the Sun" are examples of this.

By contrast, the sentences

(a) The Earth is spherical
(b) The Earth's orbit is elliptical

are not descriptive truths: empirically given Earth is not (ideally) spherical, and empirically given Earth's orbit is not a (perfect) ellipse. If (a) and (b) are true in some sense, it is only in that they indirectly, and in a simplified, idealized way, represent some empirical facts. In this case, we would usually say that (a) and (b) are approximately true.

Distinguishing descriptive and approximate truths presupposes distinguishing between descriptive and idealized reference. The former directly and without distortion relates terms such as "Earth" and "orbit" to empirically given objects, while the latter does so by idealizing them.

In light of this distinction, the question of the logical value of (a) and (b) will be ambiguous in contexts where their type of intended interpretation is not defined. So let us assume that a descriptive interpretation is defined. Then, "Earth" refers to an actual planet, "orbit" refers to real orbits, and sentences (a) and (b) will be false. Now, let us assume that an idealized interpretation is defined. Then, "Earth" refers to a perfectly spherical solid (being a geometrical idealization of a real planet), "orbit" refers to the class of ellipses (being geometrical idealizations of real orbits), and sentences (a) and (b) will be true.

A similar analysis will apply to any empirical term. In other words, any such term will be potentially ambiguous: it will have, depending on the context, a descriptive or an idealized reference.

Let us note that whenever we make deliberate use of the concept of approximate truth, we are employing an idealized interpretation, not a descriptive one. In such a context, (a) and (b) will be both referentially true *in* an idealized interpretation and approximately true *of* a (real) target system's structure (e.g., the Solar System's structure; cf. Sec. 4).

The intended interpretation and the target structure can be formally represented by models (in the sense of model theory). Thus, we will say that the sentences (a) and (b) are *referentially true in* ***m***, where ***m*** is a model representing the idealized interpretation of the constants "Earth" and "orbit," and *approximately true of* ***m'***, where ***m'*** is a model representing the real structure of the Solar System. This approximate truthfulness is made possible because ***m*** is an idealization of ***m'***.

Stathis Psillos has presented a similar conception, which he names the "intuitive approach."[14] The following quotation seems representative in this regard:

14 S. Psillos, *Scientific realism. How science tracks truth*, London–New York 1999, pp. 266–269.

> A description *D approximately fits* a state *S* (i.e. *D* is approximately true of *S*) if there is another state *S′* such that *S* and *S′* are linked by specific conditions of approximation, and *D fits S′* (*D* is true of *S′*). [...] Take, for example, the law of gases, *PV=RT*. This is approximately true of real gases, since it is true of ideal gases and the behaviour of real gases approximates that of ideal gases under certain conditions.[15]

The main novelty of the present approach compared to Psillos' clarifications is our use of the semantic notion of (descriptive or idealized) models in place of the ontological one of (real or idealized) states, and its formal reconstruction within the framework of the semantic theory of referential truth.[16]

3. Realism and antirealism about ILs

Abstract models, treated as idealizations of target structures, are commonplace in various parts of science.[17] Their effective use often leads to the formulation of scientific laws that are more or less idealized. For example, Galileo's transformations are more idealized than Lorentz's, and the Clapeyron

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15 S. Psillos, *Scientific realism*, p. 268. Psillos also accurately identifies the main philosophical cost of this solution, which is to recognize the existence of an abstract medium in the empirical representation of the world: "A price for this move is that the truth of theories does not give them straightforward representational content vis-à-vis the physical world. Their representational content is mediated (at least partly) by abstract objects—the models. Another price is that there is commitment to abstract objects—with all the (notorious) problems this move brings in tow. My view is that the price is worth paying" (S. Psillos, *Living with the abstract: Realism and models*, "Synthese" 80 (2011) issue 1, p. 9). We shall endorse this view here.

16 Our strategy for the analysis of the "intuitive approach" (in Psillos' sense) also departs from his one: Psillos holds that "here the comparison with the formal Tarskian understanding of truth is not helpful" (S. Psillos, *Scientific realism*, p. 268). It seems the author did not consider the explicative potential that lies in the semantic theory of referential truth.

17 This is especially the case in physics, as highlighted by N. Cartwright (*How the laws of physics lie*, 1983); cf. also *Models as mediators: Perspectives on natural and social science*, eds. M.S. Morgan, M. Morrison, Cambridge 1999, and many others.

equation is more idealized than the van der Waals equation. With these findings in mind, we can formulate two general postulates about ILs:

IL1 ILs are epistemically useful propositions commonly formulated in science.
IL2 Some ILs are more idealized than others.

As we know, every IL contains an idealized assumption. For instance, the ideal gas law — expressed by the Clapeyron equation — assumes that every portion of the ideal gas consists of dimensionless, spherical, and perfectly elastic molecules that exhibit no mutual attraction. Since no portions of such gas exist in the real world, this assumption is false if taken as a (direct) description of empirical facts. Both antirealists and realists commonly agree with this conclusion (cf. the final two quotations in Section 1). According to this (cf. also Section 2), we assume:

IL3 Idealized assumptions of ILs, if interpreted descriptively, are false.

The philosophical interpretation of IL3 plays an essential role in the dispute over scientific realism, where antirealists assume, unlike realists, that IL3 has critical significance. In particular, they claim that IL3, together with IL1, leads to the conclusion that falsehoods permeate all of science (cf. Section 4).

Leaving aside the question of giving a more profound philosophical interpretation of IL3, IL1–IL3 can be treated as methodologically relatively uncontroversial, meta-scientific facts. On the other hand, the following two propositions, which are typical of scientific realism, seem more difficult to treat in this way:

IL4 ILs are referential truths.
IL5 ILs are approximate truths.

As with IL3, the relevance of IL4 and IL5 to the realism/antirealism debate at least partially depends on their philosophical interpretation: whether they are to be construed in deflationary or non-deflationary terms. (In general, the *deflationary interpretation* of the thesis denies it a significant philosophical meaning.) In particular, the non-deflationary interpretation of IL1–IL5, excluding IL3, brings with it the consequence that science is permeated by truths and approximate truths, at least some of which may be the basis for future, more accurate approximations of reality. This consequence is difficult for antirealists to accept. Therefore, they have developed the non-deflationary interpretation of IL3 and have sought to attack or weaken the importance of IL4 and IL5.

In particular, antirealists have often had recourse to the deflationary interpretation of IL4, disregarding the philosophical significance of the semantic theory of truth and the underlying set theory. Among others, Cartwright expresses such a stance, declaring that "I think the formal set-theoretic apparatus would obscure rather than clarify my central points".[18] Even if she has sometimes granted ILs the status of referential truths, when so doing she points out that they are truths in models that do not reflect any essential features of the actual phenomena:

> The fundamental laws of the theory are true of the objects in the model, and they are used to derive a specific account of how these objects behave. But the objects of the model have only 'the form or appearance of things' and, in a very strong sense, not their 'substance or proper qualities'.[19]

A similar observation applies to IL5. Here are some examples of how it comes to be criticized on a deflationary approach:

> Not only do the laws of physics have exceptions; unlike biological laws, they are not even true for the most part, or approximately true.[20]

18 N. Cartwright, *How theories relate: Takeovers or partnerships?*, "Philosophia Naturalis" 35 (1998), p. 159.

19 N. Cartwright, *How the laws of physics lie*, p. 17. On the other hand, later she also writes: "Theories are true only of their models and, at best, of real systems that resemble them [the models] closely enough" (N. Cartwright, *How theories relate*, p. 33–34).

20 N. Cartwright, *How the laws of physics lie*, p. 54. Another deflationary interpretation of IL5 is recognizing that even if ILs are true, they still do not apply to target systems. Cartwright (see her first quote in Section 1) suggests this interpretation in the same context. Section 8, however, takes the

> Veritism can apparently accommodate some of the representations I have labeled felicitous falsehoods by construing them as approximations. [...] But not all felicitous falsehoods are even approximately true.[21]

> Much of science resists interpretation as successive approximation or increasingly accurate representation of phenomena, to the end of explaining and predicting those phenomena. As a result of rampant, unchecked idealization, many of the best products of science are not things we believe to be true.[22]

By *realism about ILs*, we shall mean the position expressed in the deflationary interpretation of IL3 and the non-deflationary interpretation of the other IL-related postulates (i.e., L1, L2, L4, and L5). By *antirealism about ILs*, we will understand the position expressed by the non-deflationary interpretation of IL1–IL3 and the deflationary interpretation of L4 and L5 — or even, in its strong version, their rejection.[23]

The remainder of the paper focuses on the issues of how to arrive at a proper philosophical interpretation of IL3, a rigorous, semantical justification of IL4, and a clarification of IL5. The goal here is not to comprehensively defend the realist position on ILs. Nevertheless, its aim is to demonstrate a coherent conception of empirical truth and idealization, furnishing a realistic interpretation of IL3, an acceptable justification for IL4, and a methodologically satisfactory clarification of IL5.

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opposite, realist position that ILs generally apply to target systems through a non-empty concretization relation.

21 C. Z. Elgin, *True enough*, p. 29.

22 A. Potochnik, *Idealizations and the aims of science*, Chicago 2017, p. 93.

23 A stance that differs from antirealism about ILs is that of *instrumentalists*, who argue that they are devoid of any logical value. Examples of work in which this position is advanced: N. Cartwright, T. Shomar, M. Suárez, *The tool box of science: Tools for the building of models with a superconductivity example*, Poznań 1995, pp. 137–149; B. Van Fraassen, *Scientific representation*, Oxford 2008.

4. The realist interpretation of IL3

Many antirealists about ILs have argued in recent years that epistemically useful falsehoods, most notably ILs or at least their idealized assumptions, permeate the whole of science (where a theory or a proposition is *epistemically useful* if it is an essential element of the structure of human understanding or knowledge, including scientific understanding/knowledge).[24] The basic premise underlying such statements has been aptly articulated as follows: "There are epistemically useful falsehoods that figure ineliminably in scientific understanding whose falsehood is no epistemic defect and that should be accepted".[25]

We will assume that the epistemically useful falsehoods in question are "falsehoods" in the descriptive sense of the word (and, therefore, that they are the opposite of descriptive truths).[26]

According to that position, sentence (a) ("The Earth is spherical") is false in the descriptive interpretation. So far, this part of the antirealist assessment is accurate. The problem is that not a descriptive interpretation of (a), but an idealized one, is intended in many typical scientific contexts (for example, in theoretical astronomy). In particular, its idealized interpretation is intended when (a) is consciously accepted as only approximate truth. The antirealist misinterpretation seems to consist in its reinterpreting in descriptive terms sentences that occur in contexts in which the basic intended interpretation is not a descriptive but an idealized one.

Sentences like (a) and (b) often play the role of idealized assumptions of ILs. Given the two possibilities for their interpretation, the following thesis

(*) Idealized assumptions of ILs are false,

24 This is best seen in the statements by Cartwright and Elgin quoted in Section 1.

25 T. Nawar, *Veritism refuted? Understanding, idealization, and the facts*, "Synthese" 198 (2021) issue 5, p. 4298.

26 Note that we are deliberately restricting the scope of the bearers of truth/falsehood to theories and propositions. This caveat is not self-evident: some authors (e.g., C. Z. Elgin, *True enough*) also consider non-propositional bearers of epistemic values (e.g., physical models or maps).

turns out to be ambiguous. One meaning of the term “idealized assumption” refers in this context to sentences having an intended idealized interpretation (in line with what many theoretical scientists intend). On the other hand, another refers to sentences having an intended descriptive interpretation (in accordance with the aims entertained by many philosophers). In the former instance thesis (*) is false, and in the latter case it is true.

However, the latter meaning needs to be revised: the fact that we intentionally treat a sentence as an idealized assumption alone makes us give it an idealized sense. Making such assumptions is a typical procedure in theoretical scientific contexts. Therefore, their idealized, non-descriptive interpretation is methodologically basic.

Taking into account the above analysis, the following clarification of IL3 can be proposed:

IL3’ Idealized assumptions of ILs are false in any descriptive interpretation, but no such interpretation is intended if these assumptions are intentionally made.

We encounter similar distinctions being formulated in various philosophical contexts. For instance, Arnon Levy claims that in understanding idealization the intentions of the modeler are crucial, and notes that “some authors have argued that, appropriately interpreted, many or all idealizations should not be regarded as false, or even, strictly speaking, as representing anything in the world (truly or falsely).”[27] Levy also points to the need to distinguish between the apparent (implicit) and actual (explicit) content of idealized hypotheses. If we understand “implicit content” broadly also to include metalinguistic qualifications, the actual content of sentences (a) and (b) will be expressed as follows:

(a’) In the idealized sense, the Earth is spherical.
(b’) In the idealized sense, the Earth’s orbit is elliptical.

27 A. Levy, *Idealization and abstraction: refining the distinction*, “Synthese” 198 (2018) issue 24 supplement, pp. 5855–5872.

This explication of the actual content of (a) and (b) clearly shows them to be true.

The above reasoning can be extended to ILs using a well-known argument. Assuming that "C(x)" is a predicate representing the conjunction of all its idealized assumptions and that P, V, and T are measures of pressure, volume, and temperature, respectively, the ideal gas law should be expressed in the following form:

CL For every x, if C(x), then $P(x)\cdot V(x) = n(x)\cdot R\cdot T(x)$.

Since no portion of a real gas satisfies "C(x)," every substitution of "x" with a symbol for such a portion in "C(x)" will be false. In such a descriptive interpretation (given the truth table for material implication),[28] every sentence obtained by dropping the quantifier in CL and substituting an individual constant for x will be true. Therefore, CL will not be descriptively false; on the contrary, in the *descriptive* interpretation it will be true. An analogous line of reasoning can be pursued for any IL.

To summarize, the descriptive interpretation of ILs or their assumptions implicitly assumed by antirealists is methodologically unsound. Moreover, the descriptive interpretation of ILs is ultimately unacceptable to them — if they adopt standard logic — because it leads to the consequence that ILs are true.

In light of this conclusion, IL3 turns out to be trivial (deflationary): if idealized assumptions of ILs are interpreted descriptively, they are, by definition, false. On the other hand, rejecting such an interpretation as methodologically unsound strengthens the realistic position effectively.

28 Some antirealists do clearly embrace principles of classical propositional logic: "For my purposes, classical bivalence is acceptable. Either a sentence, belief, or proposition is true or it is false" (C. Z. Elgin, *True enough*, p. 16).

5. The semantics of referential truth

Scientific laws, including ILs, are expressed in sentences that belong to the languages employed by theories. Therefore, we can reduce the question of the truth/falsehood of those laws to that of the truth/falsehood of those sentences. A systematic elaboration of this approach may lead to the conclusion that the semantic thesis stating the referential truthfulness of any law, including any IL, is simply a logical consequence of Convention T and this law's metalinguistic formulation. The remainder of this section will focus on elaborating the details of this line of thinking.

Let L be a *fully interpreted language*: i.e. a well-defined language with a complete intuitive interpretation of its logical and extralogical constants. Completeness, on this understanding, will mean that every sentence of L is well-defined content-wise, and is assigned one or other of two logical values, True or False. Let us also assume that L has a standard logical structure (i.e. that of a first-order logic with identity).

Let M_L be a *metalanguage* of L, meaning a fully interpreted language for which three conditions are met:

(i) Every L-sentence (i.e. sentence of L) is expressible in M_L;
(ii) The predicate "is true" belongs to the vocabulary of M_L;
(iii) Every L-sentence p has a name in M_L.

We then say that L is an *object language* relative to the metalanguage M_L.

A *theory* will be a set of sentential expressions formulated in a fully interpreted language, closed to the operation of logical consequence.[29]

29 The term "theory" is defined here according to the so-called Statement View. That is pretty understandable, given that in the semantic theory of truth sentences are bearers of truth and falsity. Cf. D. Portides, *Models and theories*, in: *Springer handbook of model-based science*, eds. L. Magnani, T. Bertolotti, Dordrecht–Heidelberg 2017, pp. 25–48; R.G. Winther, *The structure of scientific theories*, in: *The Stanford encyclopedia of philosophy* (2021), ed. E. Zalta, https://plato.stanford.edu/archives/spr2021/entries/structure-scientific-theories/).

Let M_L be a metalanguage containing the language of model theory, and T be a theory defined in M_L. We will say that T is a *semantic theory of truth* for L if and only if every equivalence of the form

T a_i is true in L if and only if A_i

(where A_i is a sentence of M_L expressing the content of a sentence of L denoted by the name a_i) is a thesis of T. According to the terminology adopted in that context (originating from Tarski), the above-mentioned *Convention T* will furnish a material adequacy condition for every version of this theory.[30]

There are, to be sure, many possible ways to view the semantic theory of truth.[31] In the following analysis, we shall adopt a model-theoretic one, in which the object language L is a part of the metalanguage M_L. This version of the theory will be called the *semantic theory of referential truth* — or, alternatively, the *referential truth theory* (**RTT**, for short). Within **RTT**, the univocal constants of L can be used in M_L to specify only one relational structure (model) $\boldsymbol{m}^L$ representing the intended interpretation of L. We will refer to this structure as the *intended L-model*.[32] A simple example of the construction of such a model is given below.

Let L be the language of natural-number arithmetic, with symbols for zero ("0"), one ("1"), and addition ("+"). Let N be the set of natural numbers with zero. The intended L-model will, in this case, be defined as follows:

30 Cf. W. Hodges, *Tarski's truth definitions*, in: *The Stanford encyclopedia of philosophy* (2018), ed. E. Zalta, U. Nodelman, https://plato.stanford.edu/entries/tarski-truth/.

31 From amongst these, Tarski developed two approaches. One does not use the concept of a "model". See A. Tarski, *Pojęcie prawdy w językach nauk dedukcyjnych*, Warszawa 1933. The other employs the conceptual apparatus of model theory. See A. Tarski, R. Vaught, *Arithmetical extensions of relational systems*, "Compositio Mathematica" 13 (1957) no. 2, pp. 81–102.

32 What is distinctive about the above approach is that it takes into consideration both Tarski's original assumption that the object language forms part of the metalanguage (A. Tarski, *Pojęcie prawdy w językach nauk dedukcyjnych*) and his later model-theoretic approach (A. Tarski, R. Vaught, *Arithmetical extensions of relational systems*). Let us note that this solution deviates from both the Statement View (due to the model-theoretic approach adopted) and the Semantic View (in which "theory" is defined as a class of models; cf. D. Portides, *Models and theories*, R. G. Winther, *The structure of scientific theories*).

$\boldsymbol{m}^L = \langle N, A, 0, 1\rangle$,

where A is a set of ordered triples $\langle x, y, z\rangle \in N3$ such that the following condition is met:

$\langle x, y, z\rangle \in A$ if and only if $x = y + z$.

Because the intended L-model ($\boldsymbol{m}^L$) is a formal object representing the intuitive interpretation of L,[33] the following equivalence occurs in **RTT**:

(1) For every sentence x of L: x is true in L if and only if x is true in m^L.

The standard model-theoretic definition of truth for a given language L, which we omit here, generates the relevant substitutions of T-equivalences for every sentence of L.[34] Thus, we can assume that **RTT** satisfies Convention T for any well-defined L-language.

RTT is a referential semantics in that it is based on relating the primitive terms of an object language to their corresponding constituents in the intended L-model. As we know from model theory (construed broadly),[35] this kind of semantics can be equally well applied to the formal language of mathematics, scientific theories, and well-defined parts of natural language.[36] Given these features, **RTT** fully deserves to be called the "referential truth theory."

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33 This point of view can probably be reversed so that the intended L-model can be understood as an idealized representation of a cognitive structure in the researcher's mind. In particular, $\boldsymbol{m}^L = \langle N, A, 0, 1\rangle$ can represent the cognitive structure of what goes on when adding natural numbers in the mind. In this case, L results from this structure's *syntactic formalization*.

34 Cf. J. Woleński, *The semantic theory of truth*, in: *Internet encyclopedia of philosophy* (2021), https://iep.utm.edu/s-truth/.

35 Cf. W. Hodges, *Model theory*, in: *The Stanford encyclopedia of philosophy* (2023), eds. E. Zalta, U. Nodelman, https://plato.stanford.edu/entries/model-theory/.

36 Also Suppes, as early as the 1960s., assumed that "the concept of model in the sense of Tarski may be used without distortion and as a fundamental concept […]. In this sense I would assert that the meaning of the concept of model is the same in mathematics and the empirical sciences. The difference to be found in these disciplines is to be found in their use of the concept" (P. Suppes, *A comparison of the meaning and use of models in mathematics and the empirical sciences*, in:

We can now clearly define a referential version of the semantic concepts of truth/falsehood relativized to a model:

(2) For every sentence x of L, for every model $\boldsymbol{m}$ of L: x is *referentially true/false* in the model $\boldsymbol{m}$ of L if and only if x is true/not true in L & $\boldsymbol{m}^L = \boldsymbol{m}$.

It is easy to see the following conclusion of (1) and (2):

(3) Let x be a sentence of L. The following conditions are equivalent within **RTT**:

(i) x is true in L,
(ii) x is true in $\boldsymbol{m}^L$,
(iii) x is referentially true in $\boldsymbol{m}^L$,
(iv) x is referentially true in $\boldsymbol{m}$, for some model $\boldsymbol{m}$ of L,
(v) x is not referentially false in $\boldsymbol{m}^L$.

6. How do we know that the ideal gas law is referentially true?

Our main interest is in *idealization languages* — i.e. fully interpreted languages containing empirical constants, each with an idealized reference. In particular, the language of ideal gas theory is an idealization language. If L is that language, the reconstruction of the intended L-model can take the form presented below.

Assume that the domain D of the language L contains three non-empty and disjoint sets: the set P of dimensionless and perfectly elastic gas particles, the set G of portions of the gas (composed of such particles), and the set $\mathfrak{R}$ of real numbers. Since the constants referring to the sets P and G have empirical interpretations, they will be called the *principal sets*. ($\mathfrak{R}$ will be the *auxiliary set.*)[37]

The concept and the role of the model in mathematics and natural and social sciences, ed. J. Freudenthal, Dordrecht 1961, p. 165).

37 Cf. W. Balzer, C. U. Moulines, J. Sneed, *An architectonic for science. The structuralist program*, Dordrecht–Boston 1987, pp. 1–10, where similar terminology is adopted.

Let us define the denotations of the specific function symbols "p," "V" and "T" (of the object language L) as sets of ordered pairs:

$p^* = \{<x, y> \in G \times \Re: p(x) = y\}$,
$V^* = \{<x, y> \in G \times \Re: V(x) = y\}$,
$T^* = \{<x, y> \in G \times \Re: T(x) = y\}$.

The intended model $\boldsymbol{m}^L$ for L will be a relational structure of the form

$\boldsymbol{m}^L = <D, P, G, \Re, p^*, V^*, T^*, R, \cdot, +, \ldots>$,

where R is the gas constant ($R \in \Re$) and $\cdot, +, \ldots$ are operations of multiplication, addition, etc.

The standard semantic definition of truth generates the following substitution of Convention T:

$\mathrm{T_{IG}}$ "$pV = RT$" is true in L if and only if for every $x \in G$: $p^*(x) \cdot V^*(x) = R \cdot T^*(x)$.

According to $\mathrm{T_{IG}}$, **RTT** generates the following truth condition: the ideal gas equation will be referentially true in the language of ideal gas theory if and only if, for any portion of an ideal gas, the product of its pressure and volume is equal to the product of its temperature and the gas constant R.

In recognizing the right-hand side of $\mathrm{T_{IG}}$, we must also recognize its left-hand side.

The above reasoning scheme can be deployed in any similar case. Thus, for every IL, the meta-sentence stating that IL is referentially true will be a logical consequence of the metalinguistic formulation of IL and the use of Convention T. In other words, IL4 (see Section 3) is logically inferred from the acceptance of ILs at the meta-theoretical level.

Our findings so far have not yet elucidated the meanings of the terms "descriptive interpretation" (cf. IL3) and "approximate truth" (cf. IL5). The issue arises of the expansion of **RTT** to include these and related meanings along the sort of intuitive lines outlined in Section 2. The ensuing sections will be devoted to addressing this question.

7. How to formalize descriptive truths in the Solar System model?

By analogy with the definition of idealization languages, *descriptive languages* can be defined as fully interpreted languages containing empirical constants, each with a descriptive reference. According to our initial distinctions (see Section 2), the general idea of descriptive truthfulness goes as follows:

(4) x is *descriptively true* in $\boldsymbol{m}$ of L if and only if x is true in L & L is a descriptive language & $\boldsymbol{m}^L = \boldsymbol{m}$.

If L is descriptive, model $\boldsymbol{m}^L$ directly represents a target structure (this being, in turn, a fragment of the real world's structure). We will call any model representing such a structure *descriptive*. Hence, if L is a descriptive language, $\boldsymbol{m}^L$ is a descriptive model.

Every target system has a structure represented by a descriptive model. This structure is *instantiated* in the system in the sense of the following structuralist construal:

> Although target systems are not structures, they are composed of parts that instantiate physical properties and relations. The parts can be used to define the domain of individuals, and by considering the physical properties and relations purely extensionally, we arrive at a class of extensional relations defined over that domain. This supplies the required notion of structure. We might then say that physical systems instantiate a certain structure.[38]

The Solar System can be regarded as an example of a target system. Consider the following model:

$$\boldsymbol{m}_{SS} = \langle D, P, R_1, R_2, R_3, a, b_1, \ldots, b_8 \rangle.$$

38 R. Frigg, J. Nguyen, *Modelling nature: An opinionated introduction to scientific representation*, New York 2020, pp. 74–75.

It should be noted, where the above is concerned, that:

- D is the set containing the Sun (a) and any natural object that orbits it;
- $P \subset D$ is the set of known planets that orbit the Sun;
- $R_1, R_2 \subset D^2$ are sets of ordered pairs, and $R_3 \subset D^3$ is a set of ordered triples, defined as follows:

  (i) $<x, y> \in R_1$ if and only if x orbits y,
  (ii) $<x, y> \in R_2$ if and only if x has a larger surface area than y,
  (iii) $<x, y, z> \in R_3$ if and only if x is closer to y than z;

- b_1 = Mercury, b_2 = Venus, b_3 = Earth, …, b_8 = Neptune.

The relational structure $\boldsymbol{m}_{SS}$ is a descriptive model that represents one of the structures of the Solar System. This is obvious, given that many of the basic elements of our knowledge of this system can be expressed as metalinguistic $\boldsymbol{m}_{SS}$ descriptions. Here are some examples of such descriptions:

- Earth orbits the Sun: $<b_3, a> \in R_1$.
- Earth has a larger surface area than Mercury: $<b_3, b_1> \in R_2$.
- Of the known planets in the Solar System, Neptune is farthest from the Sun: for every $x \in P$, if $x \neq b_8$, then $<x, a, b_8> \in R_3$.

The descriptions above are metalinguistic equivalents of sentences of the object language L, determined by a dictionary containing some specific, well-defined predicates ("is a natural object," "orbits," "has a larger surface area than," and "is closer than") and name constants ("Sun," "Mercury," "Venus," "Earth," etc.). Since L is a well-defined part of the metalanguage expressing these descriptions, it uniquely determines the intended L-model ($\boldsymbol{m}^L$). The following equivalence is an apparent condition for the correctness of the semantic interpretation that defines this model:

$$\boldsymbol{m}^L = \boldsymbol{m}_{SS}.$$

Similar reasoning can be pursued for any target system. Generally, any system can be represented by a descriptive model with a selected domain of the system's components and an extensional characterization that includes selected relations between them. These components and relations are defined using the corresponding constants of the object language *L*. Therefore, under such an extended **RTT**, the structure of a target system can be identified with the intended *L*-model, which is used to describe this system (or its selected aspects).

It would be a mistake to think that $\boldsymbol{m}_{SS}$ is the only structure of the Solar System; on the contrary, there are many such structures. Perhaps the simplest of them is this:

$$\boldsymbol{m}_{SS'} = \langle D, P, R_1, R_3, a, b_3 \rangle.$$

On the other hand, some natural extensions of the $\boldsymbol{m}_{SS}$ represent structures that further distinguish dwarf planets, asteroids, comets, meteoroids, and interplanetary dust, as well as rocky planets, gas giants, a collection of pairs of objects such that the first has more mass than the second, etc. Thus, the structures represented by $\boldsymbol{m}_{SS}$ and $\boldsymbol{m}_{SS'}$ are only two of the many structures of the Solar System that are determined by more or less fine-grained languages serving the purpose of their description.

We assume that each target system has multiple structures, and that some descriptive models uniquely represent them.[39] Moreover, each target system, in combination with its description language, will uniquely determine its structure.

We can now return to our preliminary idea of descriptive truth/falsehood. If *L* is a language for describing the Solar System such that $\boldsymbol{m}^L = \boldsymbol{m}_{SS}$, then the following sentences are descriptively true in $\boldsymbol{m}^L$: "Earth orbits the Sun," "Earth has a larger surface area than Mercury," and "Of the known planets in the Solar System, Neptune is farthest from the Sun."

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39 The argument that the target system does not uniquely determine its structure has been one of the main objections to the structuralist theory of representation (cf. R. Frigg, *Scientific representation and the semantic view of theories*, "Theoria" 21 [2006], pp. 49–65). An analysis of this objection leads to the conclusion that "[s]ystems only have a structure under a particular description, and there are many non-equivalent descriptions" (R. Frigg, J. Nguyen, *Modelling nature*, p. 76). As we see, fully interpreted object languages can play the role of such descriptions within **RTT**.

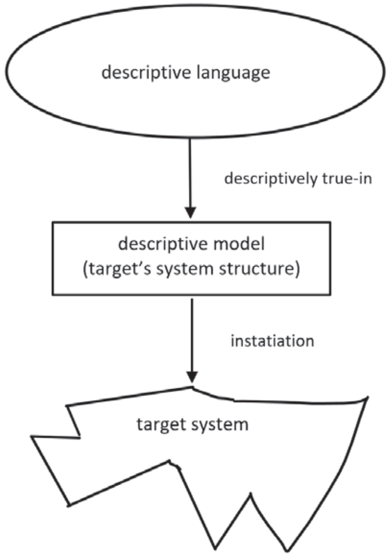
Note that the following condition is a simple consequence of our clarifications:

(4) For every *x* of *L*: if *L* is not descriptive, then *x* is neither descriptively true in *L* nor descriptively false in *L*.

Consequence (4) precludes the possibility of any IL, if interpreted properly (cf. Section 3), being a descriptive truth or a descriptive falsehood.

The following figure shows the relations discussed above:

Fig. 1



8. The concretization relation and approximate truth

A sentence is approximately true of a target structure if and only if it is true in the intended model, which is an idealization of this structure (cf. Section 2). In particular, every IL can be treated as an example of approximate truth. The task of this section is to explicate this idea within the **RTT** framework.

Here we will treat the intended L-model, where L is an idealization language, as an abstract, *idealized model* that represents a descriptive structure in a simplified way. For instance, in the idealized model of the physical pendulum, the abstract (non-stretchable and massless) thread of the pendulum represents the real (and therefore stretchable and mass-possessing) thread. In an idealized gas model, the abstract (dimensionless and perfectly elastic) particles represent the real particles of the gas. With this approach in mind, we will try to clarify the notion of "idealization" as used when describing the representation relation that obtains between idealized and descriptive (target) structures.[40]

As usual, we will assume that the idealization has its converse, this being the de-idealization relation. However, in a departure from what is typical we will assume that it is not the former but the latter that is logically more primary within the framework of an expanded **RTT**.

More specifically, we will assume that any idealized model $\boldsymbol{m}^L$ can be concretized in a descriptive structure $\boldsymbol{m}^S$ of a target system S. This concretization will sometimes be empty. If it is non-empty, $\boldsymbol{m}^L$ will be an idealization of $\boldsymbol{m}^S$.

We can trace more closely how a descriptive structure can concretize an idealized model (uniquely determined by an idealization language). Let S be a thermodynamic system consisting of molecules and portions of a real gas. If L is a language of ideal gas theory, we can adequately modify its intended interpretation $\boldsymbol{m}^L$ (cf. Section 6) and obtain the following structure:

$$\boldsymbol{m}^S = \langle D^S, P^S, G^S, \mathfrak{R}, p^{S*}, V^{S*}, T^{S*}, R, \cdot, +, \ldots\rangle,$$

where:

- P^S is the set of all molecules in S (thus, the idealized assumption that gas molecules are dimensionless and perfectly elastic is dropped here),

40 R. Frigg (*Scientific representation and the semantic view of theories*) refers to this problem as the "enigma of representation", while Frigg and Nguyen (*Modelling nature*) call it the "Scientific Representation Problem". The latter provides an extensive discussion and critique of recent attempts to address this issue, while the approach presented below does not replicate any of the latter. Because of its use of two key concepts, "relational structure" and "instantiation of a structure" (see below), the structuralist conception is the closest to an expanded **RTT**.

- G^S is the set of all portions of the gas in S,
- D^S is the sum of the sets P^S, G^S, and $\mathfrak{R}$,
- $p^{S*}(x)$ is the result of correctly measuring the pressure of x (where x is a portion of the measured gas in S),
- $V^{S*}(x)$ is the result of correctly measuring the volume of x,
- $T^{S*}(x)$ is the result of correctly measuring the temperature of x.

In general, the *concretization* of $\boldsymbol{m}^L$ in S (where L is an idealization language, and S is a target system) is a relational structure (model) that is the result of replacing each principal set in $\boldsymbol{m}^L$ with a corresponding set of empirically available objects in S and redefining the relations and functions on these sets accordingly. (Sometimes, we refer to the principal sets that make up a given model as "being concretized.")

As mentioned, the set of real-world objects corresponding to the principal set of an idealized model may be empty. For example, if S is a system containing no gas particles, then the concretizations of P and G (the principal sets in the ideal gas model) in S will be empty. Similarly, if S is a (real) system of light propagation, then the concretization of the idealized set of ether particles in S will be empty. In each such case, we will say that the concretization of the idealized model is *empty*. The structure $\boldsymbol{m}^L$ is an idealization of $\boldsymbol{m}^S$ if and only if the concretized model $\boldsymbol{m}^S$ is non-empty.

Now, Clapeyron's equation may be said to be referentially true in $\boldsymbol{m}^L$ and, simultaneously, true approximately of $\boldsymbol{m}^S$, in that $\boldsymbol{m}^L$ is an idealization of $\boldsymbol{m}^S$. (From the empirical point of view, this is evidenced by the results of the measurements of pressure, volume, and temperature of the gas portion under test, which differ only marginally from the values predicted by the equation.)

This raises the question of whether the approximate truth predicate should be relativized to the variable S (e.g., in the form: "x is approximately true of the S-structure $\boldsymbol{m}$"). Given the semantic framework of considerations adopted here, such an extension in the direction of the ontology of systems could be theoretically cumbersome. To preserve the semantic uniformity of the approach, we will limit ourselves to the domain of model theory (although we will give some general indications as to the possible manner of such an extension — see Section 9).

Such an approach is not only possible, but also natural. This becomes apparent when we consider that in every case of using an idealized model, it is clear to which target system this model is referred. Thus, it can be assumed that each such use designates the target system first, and is part of the proper idealized model concretization. According to this approach, the full concretization of an idealized model $\boldsymbol{m}^L$ in practice consists of two steps: (i) determining the target system (based on the domain of $\boldsymbol{m}^L$) and (ii) concretizing $\boldsymbol{m}^L$ in that system (based on the characteristics of $\boldsymbol{m}^L$).

Thus, the practical feasibility of concretization procedures allows us to treat the dyadic predicate "is a concretization of" as a primary meta-term of the expanded **RTT** referring to a relation between models. Here are our basic formal findings about it:

(5) For every ***m***, for every ***m'***: if ***m*** is a concretization of ***m'***, then ***m*** is a descriptive model and ***m'*** is an idealized model.
(6) For every ***m***, for every ***m'***: ***m*** is an idealization of ***m'*** if and only if ***m'*** is a non-empty concretization of ***m***.
(7) For every sentence *x* of *L*, for every model ***m*** of *L*: *x* is approximately true of ***m*** in *L* if and only if *x* is true in *L* and *L* is an idealization language and $\boldsymbol{m}^L$ is an idealization of ***m***.

The following explication essentially concludes our analysis:

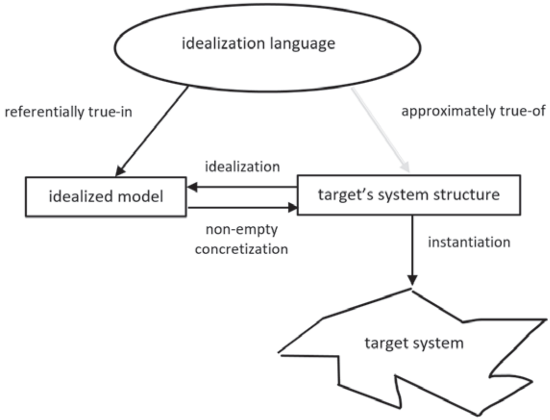
(8) *x* is *empirically true of* some ***m***-structure in *L* if and only if *x* is true in *L* and one of the following conditions is met:

- *L* is a descriptive language & $\boldsymbol{m}^L = \boldsymbol{m}$, or
- *L* is an idealization language & $\boldsymbol{m}^L$ is an idealization of ***m***.

Consequently, a sentence *x* will be empirically true of a structure ***m*** (in *L*) if *x* is either descriptively true in *m* or *x* is approximately true of ***m***.

The following figure shows the relations discussed above:

Fig. 2.



9. Concluding remarks

Extending the referential version of the semantic theory of truth with the distinction between descriptive and idealization languages and the concept of the concretization relation has made it possible for us to distinguish precisely between descriptive and approximate kinds of empirical truth. The extension also sheds new light on the debate around realism about ILs. In particular, it supports the realistic interpretation of idealized assumptions (Section 3), justifies the claim that idealized laws are referential truths (Section 6), and clarifies the belief that they are approximate truths (Section 8).

While the present paper does not aim to expand this conception any further, a theoretical perspective that includes the notion of target system representation can be sketched here in general terms. The following definition suggests this:

(9) ***m*** *represents S* if and only if for some ***m'***, ***m*** is an idealization of ***m'*** & ***m'*** is instantiated in *S*.

The representation relation, as defined in (9), seems to satisfy the list of adequacy conditions presented by Frigg and Nguyen:

1. *Surrogative reasoning condition* (models represent their targets in a way that allows us to generate hypotheses about them).
2. *Possibility of misrepresentation* (if $\boldsymbol{m}$ does not accurately represent S, then it is a misrepresentation but not a nonrepresentation).
3. *Targetless models* (what are we to make of scientific representations that lack targets?).
4. *Requirement of directionality* (models are about their targets, but targets are not about their models).
5. *Applicability of mathematics condition* (how the mathematical apparatus used in $\boldsymbol{m}$ latches onto the physical world).[41]

Ad 1. Definition (9) satisfies this condition more or less by definition, given the semantic account of the representation problem.

Ad 2. An idealized model $\boldsymbol{m}^L$ misrepresents a target system S if the concretization of $\boldsymbol{m}^L$ in a structure $\boldsymbol{m}^S$ instantiated in S is empty (even if some of the principal sets in $\boldsymbol{m}^L$ have non-empty concretizations in this structure).

Ad 3. This condition will either amount to the same as the previous one or to a strengthening of it — if we assume that "lack of a target system" means a situation in which all concretizations of principal sets are empty.

Ad 4. As in case 1, the fulfillment of this condition is quite evident from the standpoint of a semantic view of the problem of representation and is due to the intentional (i.e. in a sense "directional") nature of language.

Ad 5. Like other components of $\boldsymbol{m}^L$, certain parts of the mathematical apparatus are abstract components of the representation of the system S. The mathematical apparatus embedded in $\boldsymbol{m}^L$ relates to the target system S (among other things, through the "measurement" part of the structure $\boldsymbol{m}^S$) as a part of the abstract representation of the structure $\boldsymbol{m}^L$.

41 R. Frigg, J. Nguyen, *Models and representation*, in: *Springer handbook of model-based science*, p. 55.

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